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EFFECTS OF BOUNDARY CONDITIONS AND INITIAL OUT-OF-ROUNDNESS ON THE STRENGTH OF THIN-WALLED CYLINDERS SUBJECT TO EXTERNAL HYDROSTATIC PRESSURE

by

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Effects of Boundary Conditions and Initial Out-of-Roundness on the Strength of Thin-Walled Cylinders Subject to External Hydrostatic Pressure

By G. D. GALLETLY[1] and R. BART[2]

Using classical small-deflection theory, an investigation was made of the effects of boundary conditions and initial out-of-roundness on the strength of cylinders subject to external hydrostatic pressure. The equations developed in this paper for initially out-of-round cylinders with clamped ends, and a slightly modified form of the equations previously derived by Bodner and Berks for simply supported ends, were applied to some actual test results obtained from nine steel cylinders which had been subjected to external hydrostatic pressure. Three semiempirical methods for determining the initial out-of-roundness of the cylinders also were investigated and these are described in the paper. The investigation indicates that if the initial out-of-roundness is determined in a manner similar to that suggested by Holt then the correlation between the experimental and theoretical results is quite good. The investigation also indicates that while the difference in collapse pressures for clamped-end and simply supported perfect cylinders may be quite considerable, this does not appear to be the case when initial out-of-roundnesses of a practical magnitude are considered.

Introduction

Several analyses have appeared in the literature for the elastic buckling of a thin cylindrical shell subject to external hydrostatic pressure (1)[3] The majority of these analyses have been based upon the classical small-deformation theory of thin shells and have assumed a geometrically perfect, stress-free structure prior to loading. The correlation obtained between these theories and experimental results has been good for long cylinders but rather poor for short cylinders. Efforts are currently being made by several investigators to explain the discrepancy between theory and experiment in the short-cylinder range by the use of large-deflection theory. At this date, however, it is not known by how much this discrepancy will be reduced as all the final reports on their work have not been published.

One possible cause for the discrepancy between theoretical and experimental results can be ascribed to the initial out-of-roundness of the cylinders, and a number of investigations, using small-deflection theory, already have been made on the effect of initial irregularities on the collapse pressure of cylinders subject to external hydrostatic pressure (2, 3, 4). As was to be expected, these analyses showed that the initial irregularities reduced the failure pressures below those of the perfect cylinders. However, when these analyses were applied to some models which had been tested experimentally, they predicted failure pressures which were less than three quarters of those observed experimentally. Since these analyses had assumed simple supports at the ends of the cylinders and it was probable that the boundary conditions of the models were somewhere between the extremes of simple supports and clamped ends, it was of interest to investigate the reduction in collapse pressure of clamped-end cylinders due to initial irregularities, to see if the assumed boundary conditions could explain the discrepancy between experiment and theory. Also, the analyses assume that the initial out-of-roundness in the cylinders is similar to one of the modes into which a perfectly circular cylinder of the same dimensions would buckle, and actual shells never satisfy this condition. It thus seemed desirable to investigate the various simplified methods that have been suggested for determining the initial out-of-roundness of the cylinders to see what effect these had upon the computed failure pressure. These methods are described in the paper.

It is also of interest to note that there are other limitations in the existing linearized theories. These are:

(a) The fact that in these problems there usually exist other buckling pressures close to the minimum buckling pressure. Thus, use of only that mode of initial out-of-roundness which corresponds to the minimum buckling pressure is really only defensible at pressures very close to the minimum buckling pressure.

(b) The simple yield criterion used to predict failure.

This point is discussed under the section entitled "Assumptions Made in Analysis."

The authors have not investigated the foregoing factors but hope to do so in the future.

The approach used in this paper is similar to that of Bodner and Berks (3), except that instead of using a Donnell-type equation Galerkin's method was employed in conjunction with a modified Donnell-type equation. The initial out-of-roundness pattern assumed by Bodner and Berks was of the form

$$w_0 = e \sin m\theta \cos \frac{\pi x}{L}$$

(origin at mid-length) while that assumed by the present authors was

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$$w_0 = \frac{e}{2}\sin m\theta\left[1-\cos\frac{2\pi x}{L}\right]$$

(origin at one end of the cylinder) Thus, in both cases, the initial out-of-roundness satisfied the relevant boundary conditions and was also similar in form to one of the assumed buckling modes. The magnitude of the initial out-of-roundness at the ends and center of the cylinder was also the same in both cases.

The Bodner-Berks solution and that presented herein thus represent lower and upper bounds for the effect of initial eccentricities on the collapse pressures of elastically supported unstiffened cylinders, when the initial eccentricities have the same shape as one of the assumed buckling modes of the perfect cylinder. While the two solutions are not exact, they should provide good approximations to the exact solutions. One of the limitations of using Donnell's equation is that the number of circumferential lobes should be fairly high, and thus the results will be slightly in error for very long cylinders which buckle into two or three circumferential lobes.

The final results of the investigation are given in Fig 2 and in Tables 2, 4, and 5. It can be seen that the correlation between experiment and theory is quite good when method (c) is used to determine the initial out-of-roundness of the models. [Method (c) is similar to that suggested by Holt (5).] However, it is not claimed that the results give a complete answer to the problem and more work of both an experimental and theoretical nature is required

Method of Analysis

The modifications to Donnell's equation brought about by initial eccentricities in the shell have been presented by Bodner and Berks (3), and, prior to them, by Cicala (6). The equations also have been derived by the authors in the Appendix, using a somewhat different approach to that adopted by the previously mentioned authors For the case of uniform external hydrostatic pressure applied on all sides of an imperfect cylinder the relevant equations are, from Equations [19a], [19b], [21], and [24] in the Appendix

$$\left.\begin{array}{ll}
D\nabla^4 w + \dfrac{Eh}{R^2}\nabla^{-4}w_{xxxx} + pR\left[1/2(w+w_0)_{xx} + \dfrac{1}{R^2}(w+w_0)_{\theta\theta}\right] - p = 0 \ldots & (a)\\
\nabla^4 u = \dfrac{1}{R}\left[\nu w_{xxx} - \dfrac{1}{R^2}w_{x\theta\theta}\right]. & (b)\\
\nabla^4 v = \dfrac{1}{R^2}\left[(2+\nu)w_{xx\theta} + \dfrac{1}{R^2}w_{\theta\theta\theta}\right] & (c)\\
\nabla^4 F = -\dfrac{E}{R}w_{xx} \quad \ldots & (d)
\end{array}\right\} \quad [1]$$

where

h = thickness of shell
R = mean radius of shell
p = applied hydrostatic pressure
E = modulus of elasticity
D = $Eh^3/12(1-\nu^2)$
ν = Poisson's ratio
F = stress function of the total membrane stresses
$\nabla^4 = \left(\frac{\partial^2}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2}{\partial\theta^2}\right)^2$, $\nabla^4\nabla^{-4}f = f$
w_0 = initial radial out-of-roundness (+ inwards)
u, v, w = elastic axial, tangential, and radial (+ inwards) displacements of the imperfect cylinder minus the uniform compression experienced by a perfect cylinder (see Equation [18] in the Appendix).

The subscripts x and θ indicate partial differentiations with respect to those variables.

The patterns assumed for w and w_0, and which satisfy the boundary conditions for clamped-end cylinders, were as follows

$$\left.\begin{array}{l}
w = B\sin m\theta\left[1-\cos\dfrac{2\pi x}{L}\right]\\
w_0 = \dfrac{e}{2}\sin m\theta\left[1-\cos\dfrac{2\pi x}{L}\right]
\end{array}\right\} \ldots\ldots\ldots [2]$$

where

B = half amplitude of w-displacement
e = maximum value of initial radial out-of-roundness
m = number of circumferential waves
L = unsupported length of shell
x, θ = axial and angular co-ordinates

If the Expressions [2] happen to be an exact solution of the problem, then they will satisfy the differential equation of equilibrium, Equation [1c], exactly. However, as both w and w_0 were chosen to satisfy the boundary conditions rather than the equilibrium equation, this, in general, will not be the case. The resulting expression will be a function of x and θ which we shall denote by Q. Galerkin's equation for determining the relations between the coefficients B and e is then

$$\int_0^{2\pi}\int_0^L Q\sin i\theta\left[1-\cos\frac{2\pi x}{L}\right]Rd\theta dx = 0 \ldots\ldots [3]$$

where i assumes the values $1, 2, 3 \ldots\ldots$

For $i \neq m$ Equation [3] will be found to be zero identically For $i = m$ the following relation between B and e, obtained from Equation [3], will be found to hold

$$B = \frac{e}{2}\,\frac{p}{p_{cr}-p} \quad \ldots\ldots\ldots [4]$$

where p_{cr} is given by the expression

$$p_{cr} = \frac{E\left\{\dfrac{\left(\dfrac{h}{R}\right)^2[3m^4+2m^2\Delta^2+\Delta^4]}{12(1-\nu^2)} + \dfrac{\left(\dfrac{h}{R}\right)\Delta^4}{(m^2+\Delta^2)^2}\right\}}{(3m^2+1/2\Delta^2)} \quad \ldots [5]$$

and

$$\Delta = \frac{2\pi R}{L}$$

The smallest value of the buckling pressure of the perfect cylinder p_{cr} is found by minimizing Equation [5] with respect to m. A relation similar to that expressed by Equation [5] has recently been presented by Nash (7) using an energy method. A relation similar to Equation [4] was also obtained by Bodner and Berks for simply supported imperfect cylinders.

Thus, from Equations [2] and [4], we obtain the following expression for w

$$w = \frac{e}{2}\,\frac{p}{p_{cr}-p}\sin m\theta\left[1-\cos\frac{2\pi x}{L}\right]. \quad [6]$$

The bending moments in the shell can then be calculated from the relations

$$
\left.\begin{aligned}
M_x &= -D\left[w_{xx} + \frac{\nu}{R^2} w_{\theta\theta}\right] \\
M_\theta &= -D\left[\frac{1}{R^2} w_{\theta\theta} + \nu w_{xx}\right]
\end{aligned}\right\} \quad [7]
$$

The maximum bending stresses are then given by

$$
\sigma_{bx} = \pm \frac{6}{h^2}(M_x)_{max}, \qquad \sigma_{b\theta} = \pm \frac{6}{h^2}(M_\theta)_{max} \quad [8]
$$

To obtain the total normal stresses we now have to add the membrane stresses to Equation [8]. To determine these latter we solve Equations [4] and [6] for the stress function F (periodic terms only). The total membrane stresses are then given by

$$
\sigma_{mx} = -\frac{pR}{2h} + \frac{F_{\theta\theta}}{R^2}; \qquad \sigma_{m\theta} = -\frac{pR}{h} + F_{xx} \quad [9]
$$

The total normal stresses are obtained by adding algebraically Equations [8] and [9]. The greatest normal stresses occur at mid-length of the cylinder ($x = L/2$) and where $\sin m\theta = \pm 1$ (trough and crest points of the lobes). At these points the twisting moment $M_{x\theta}$ is zero and thus the normal stresses are principal stresses. The absolute maximum normal stresses occur at the outer shell wall for the trough points.

Having obtained the maximum principal stresses σ_x and σ_θ in terms of the initial out-of-roundness, the geometric parameters and the applied external hydrostatic pressure, we now employ the octahedral shear-stress criterion of failure (which gives the same results as the Hencky-von Mises criterion of failure), viz.

$$
\sigma_y^2 = \sigma_\theta^2 + \sigma_x^2 - \sigma_\theta\sigma_x \quad [10]
$$

where σ_y is the yield point of the material. Substitution of the maximum principal stresses σ_x and σ_θ in Equation [10] then gives an equation relating the initial out-of-roundness, the geometric parameters of the shell, the yield point of the material and the pressure at which the shell begins to yield p_y.

It should be noted that instead of using the yielding criterion given by Equation [10] where σ_x and σ_θ are principal stresses, it is more accurate to use the expression

$$
\sigma_y^2 = \sigma_\theta^2 + \sigma_x^2 - \sigma_\theta\sigma_x + 3\tau_{x\theta}^2 \quad [10a]
$$

where now σ_x, σ_θ, and $\tau_{x\theta}$ are the normal and shear stresses at any point and which are functions of x and θ. Yielding will first occur in the cylinder for those values of x and θ which maximize the right-hand side of Equation [10a]. However, to compute these values of x and θ by differentiation of equation [10a] involves more complications than seem warranted. Trials indicate that the stress condition at the outer shell wall for trough points of a lobe is probably as unfavorable as anywhere else. As mentioned earlier the twisting moment $M_{x\theta}$ is zero at these points and thus Equation [10a] reduces to Equation [10].

As we shall later present curves of p_y, the pressure at which shell yielding commences, versus e/h, the initial eccentricity-shell thickness ratio, for both simply supported and clamped-end cylinders, we have summarized the results obtained in this paper and those obtained by Bodner and Berks in Table 1. (We have added a few terms to the latter solution, as Bodner and Berks neglected the periodic terms in Equation [9]). Also, we used the expression $w_{\theta\theta}/R^2$ for the circumferential change in curvature instead of $(w_{\theta\theta}/R^2 + v_\theta/R)$ which was used by Bodner and Berks when computing the bending stresses due to initial out-of-roundness. Our expression is consistent with the expression used in deriving the approximate equations of equilibrium for an initially out-of-round cylinder (see Appendix) and also is the same as that used by Donnell in his work on perfect cylinders (8). The effect of using $w_{\theta\theta}/R^2$ instead of $(w_{\theta\theta}/R^2 + v_\theta/R)$ is to eliminate the quantity f which appears in the equations developed by Bodner and Berks.) It can be seen that the final equation (Equation [12] in Table 1) relating the initial out-of-roundness and the pressure to cause first yielding is essentially the same for both clamped-end and simply supported cylinders, except for the factor of 2 required by the definition of e as the maximum initial out-of-roundness and the slight changes in definition of the quantities K and β appearing in that equation.

TABLE 1 COMPARISON OF EQUATIONS AND PARAMETERS FOR SIMPLY SUPPORTED AND CLAMPED-END CYLINDERS HAVING AN INITIAL OUT-OF-ROUNDNESS SIMILAR IN SHAPE TO ONE OF THE ASSUMED BUCKLING MODES

	Simple supports	Clamped ends	Equation no.
Assumed w and w_0	$w = A \sin m\theta \cos \frac{\pi X}{L}$; $w_0 = e \sin m\theta \cos \frac{\pi x}{L}$ (Origin at mid-length)	$w = B \sin m\theta\left[1 - \cos\frac{2\pi x}{L}\right]$; $w_0 = e/2 \sin m\theta\left[1 - \cos\frac{2\pi x}{L}\right]$ (Origin at one end)	
p_{cr}	$\dfrac{E\left\{\dfrac{\left(\frac{h}{R}\right)^2[m^2+\delta^2]^2}{12(1-\nu^2)} + \dfrac{\left(\frac{h}{R}\right)\delta^4}{[m^2+\delta^2]^2}\right\}}{\left[m^2 + \frac{1}{2}\delta^2\right]}$; $\delta = \frac{\pi R}{L}$	$\dfrac{E\left\{\dfrac{\left(\frac{h}{R}\right)^2[3m^4+2m^2\Delta^2+\Delta^4]}{12(1-\nu^2)} + \dfrac{\left(\frac{h}{R}\right)\Delta^4}{[m^2+\Delta^2]^2}\right\}}{\left[3m^2 + \frac{1}{2}\Delta^2\right]}$; $\Delta = \frac{2\pi R}{L}$	[11]
Equation relating p_y and e	$\frac{e}{h} = \left(1 - \frac{p_y}{p_{cr}}\right)\left\{\frac{-3 \pm [9 - 4(1-\beta+\beta^2)H]^{1/2}}{4(1-\beta+\beta^2)K}\right\} \times \begin{cases}1 & \text{for simple supports}\\ 2 & \text{for clamped ends}\end{cases}$		[12]
$\frac{2p_{cr}(1-\nu^2)}{E}\left(\frac{R}{h}\right)^2 K$	$m^2 + \nu\delta^2 + 2(1-\nu^2)\frac{R}{h}\frac{\delta^4}{[m^2+\delta^2]^2}$	$2m^2 - 1 + \nu\Delta^2 + 2(1-\nu^2)\frac{R}{h}\frac{\Delta^4}{[m^2+\Delta^2]^2}$	
β	$\left\{\dfrac{\nu m^2 + \delta^2 + 2(1-\nu^2)\frac{R}{h}\frac{m^2\delta^2}{[m^2+\delta^2]^2}}{m^2 + \nu\delta^2 + 2(1-\nu^2)\frac{R}{h}\frac{\delta^4}{[m^2+\delta^2]^2}}\right\}$	$\left\{\dfrac{\nu[2m^2-1] + \Delta^2 + 2(1-\nu^2)\frac{R}{h}\frac{m^2\Delta^2}{[m^2+\Delta^2]^2}}{2m^2 - 1 + \nu\Delta^2 + 2(1-\nu^2)\frac{R}{h}\frac{\Delta^4}{[m^2+\Delta^2]^2}}\right\}$	
H	$\left[3 - \left(\frac{2h\sigma_y}{Rp_y}\right)^2\right]$		

Assumptions Made in Analysis

Besides the approximate strain and change in curvature-displacement relations used in the derivation of the Donnell equation for an initially out-of-round cylinder, and the assumption that it is initially stress free, it might be useful to list some of the other assumptions that have been made both in the analysis of this paper and that of Bodner and Berks. These are as follows:

1 That the circumferential membrane stress in the shell is constant along its length and equal to $-pR/h$, whereas this is actually not the case.

2 The assumption that the initial out-of-roundness is small (with an order of magnitude of one shell thickness) is symmetric with respect to the center line of the shell, and has the same form as one of the buckling modes of a perfect cylinder with the same shell dimensions. Actual shells rarely, if ever, satisfy the last two requirements and so the question arises as to how the initial out-of-roundness should be measured.

3 The assumption that failure occurs (appearance of visible lobes) when the most highly stressed points in the cylinder start to yield. Actually, failure does not occur until plastic regions form at the trough and crest points of the lobes. The pressure required to produce these yield zones is greater than that at which the most highly stressed points begin to yield and neglect of this effect therefore underestimates the strength of the shells. An adequate theory to take this effect into account has not been developed as yet, but, as for beams, presumably the ratio of the pressure to cause first yielding to the pressure required for the formation of plastic regions depends on the relative magnitudes of the direct stresses in the cylinder wall and the bending stresses resulting from initial out-of-roundness.

4 The assumption that Poisson's ratio is a constant and equals 0.3.

As it is intended to apply the analyses developed for unstiffened cylinders to stiffened cylinders which failed by buckling between ring stiffeners, it would be well if we enumerated the additional assumptions that were made. These are:

5 That the stiffening rings at the ends of any bay are perfectly circular, i.e., they do not have any initial out-of-roundness. This never occurs in practice, of course, but should not be too serious if the circularity of the stiffening rings is very much better than that of the shell, or if the predominant mode of initial out-of-roundness in the rings is very different from the predominant mode in the shell.

6 As for unstiffened cylinders, the circumferential membrane stress in the perfect cylinder is assumed to be $-pR/h$, whereas it actually varies along the length of the shell. A more correct representation of the stress distribution would be obtained by using the analysis of von Sanden and Günther (9), or more accurately still, that of Salerno and Pulos (10).

Methods of Determining Initial Out-of-Roundness

As mentioned hitherto, the analyses assume that the initial out-of-roundness is symmetrical about the mid-length of the shell and that its circumferential variation is in the shape of one of the buckling modes of a perfect cylinder. Actual shells do not meet either of these requirements. If we do not make a harmonic analysis of the initial out-of-roundness, and also extend the theory to account for the various harmonic components, the question arises as to how we shall measure the quantity e, which is defined in the analysis as the maximum initial out-of-roundness when its shape is similar to one of the buckling modes. As far as the authors are aware three simplified, semiempirical methods for determining the initial out-of-roundness have been proposed in the literature so far. These are (see Fig. 1):

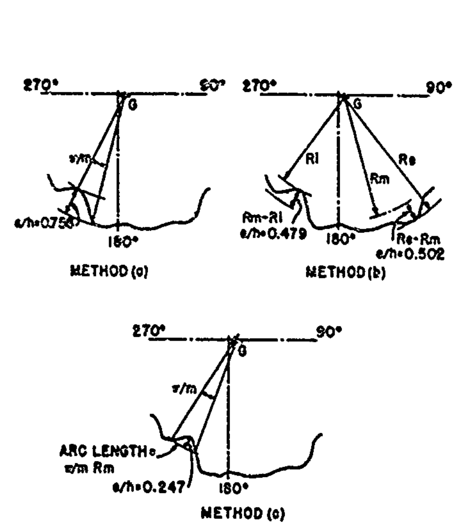

Fig. 1 Illustrations of Three Methods for Determining Initial Out-of-Roundness of Cylinders at Station 4 of Model BR-6

(a) The centroid of the initial circularity contour is first determined. Then the angle π/m, where m is the number of lobes into which the perfect cylinder would buckle, is calculated. A sector of a circle, subtending this angle π/m is then drawn on transparent paper and placed with its apex at the centroid of the initial circularity contour. The sector is then rotated so that it traverses the entire circumference of the circularity contour until the location is found at which the maximum difference between the two sector radii occurs. The initial out-of-roundness is then taken as this maximum difference. This method of determining the initial out-of-roundness is essentially that proposed by Saunders, Trilling, and Windenburg (11, 12).

(b) Both the centroid and the area of the initial circularity contour are determined. The radius R_m of the circle whose area is the same as that of the initial circularity contour is then determined. A circle, with center at the centroid of the initial circularity contour and of radius R_m, is then drawn. The initial out-of-roundness is then taken as the maximum value of $[R_e - R_m]$ and $[R_m - R_i]$, where R_e and R_i are the radius vectors from the centroid to points on the initial circularity contour which are exterior and interior, respectively, to the circle of radius R_m. A method for determining the initial out-of-roundness similar to the foregoing has been suggested, among others, by Bodner and Berks (3).

(c) As in (b) both the centroid of the initial circularity contour and the radius R_m of the mean circle are determined. Also, as in (a), the angle π/m is calculated. The arc length of one half-lobe is then obtained as $(\pi/m)R_m$. This arc is then moved around the initial circularity contour with its end points always in contact with the contour. The initial out-of-roundness is then taken as the maximum radial distance between the circularity contour and the arc. This method for determining the initial out-of-roundness is somewhat similar to the method proposed by Holt (5).

Numerical Results

In this section we present the results obtained by applying the analysis developed for simply supported imperfect cylinders (3), and its extension to clamped ends, to ... steel welded cylinders that have been tested at the Taylor Model Basin. At failure, all these models had lobes which partially covered the circumference

TABLE 2 GEOMETRIC RATIOS, YIELD POINTS, EXPERIMENTAL AND THEORETICAL COLLAPSE PRESSURES FOR STEEL CYLINDERS

(1) Model no.	(2) $\frac{L}{2R}$	(3) $\frac{h}{2R}$	(4) σ_y (psi)	(5) Number of stiffeners	(6) Expt. buckling pressure, psi	(7) Maximum sustaining pressure, psi	(8) Theoretical elastic buckling pressure from Eq. [11] of Table 1, psi: Simple supports	(9) Clamped ends	(10) Pressure to cause axisymmetric yielding of stiffened cylinders[c]
50	0 125	0 0028	39000	2 internal	175	195 (18, 19)[a]	301 (17)	451 5 (18)	264
33	0 250	0 0032	31000	2 internal	133	139 (13, 14)	192.5 (12)	262 (14)	225
66	0.500	0 0048	40000	none	235[b]	235 (8, 9)	253 (8)	395 (10)	..
71	0 500	0 0065	44000	none	327[b]	327 (7)	556 (8)	804 (9)	...
42	1.000	0 0033	43000	none	58	59 (8)	51.2 (7)	73.4 (8)	...
61	2.000	0 0040	29000	none	48[b]	48 (5, 6)	41.1 (5)	60.2 (6)	...
BR-1	0 184	0 0024	61700	5 external	80	107 (15)	135 (15)	183 (17)	322
BR-4	0.250	0.0049	50600	5 external	390	290 (10)	613 (11)	848 (12)	550
BR-5	0 184	0 0023	54400	5 external	80	95 (14)	120 (15)	165 (17)	284

[a] Numbers in parentheses are the number of lobes at failure.
[b] No distinction made in Windenburg's notes between buckling and maximum sustaining pressures for these models, hence, they were assumed to be equal in the above table.
[c] Calculated for median surface of shell at mid-bay length using analysis of von Sanden and Günther in conjunction with octahedral shear stress criterion

TABLE 3 MAXIMUM e/h-VALUES OBTAINED BY DIFFERENT METHODS FOR DETERMINING INITIAL OUT-OF-ROUNDNESS

(1) Model no.	(2) Method (a) C.E.	Method (a) S.S.	(3) Method (b)	(4) Method (c) (i) Outward C.E.	(i) Outward S.S.	(5) Method (c) (ii) Inward C.E.	(ii) Inward S.S.	(6) Method (c) Max. of (i) and (ii) C.E.	Max. of (i) and (ii) S.S.
50	0 186[a]	0.140	0.175	0.016	0.017	0.074	0.073	0.074	0.078
33	0.097	0.104	0.098	0.067	0.078	0.085	0.093	0.085	0.093
66	0.043	0.047	0 041	0.041	0.045	0.032	0.037	0.041	0.045
71	0 139	0.143	0.118	0.008	0.011	0.090	0.093	0.090	0.093
42	0.267	0.272	0 191	0.080	0.091	0.114	0.121	0.114	0.121
61	0 125	0 130	0.062	0 027	0.033	0.022	0.027	0.027	0.033
BR-1 [6, 8][b]	0 575(6)[b]	0 615(8)	0 554(2)	0.319(6)	0 375(6)	0 287(6)	0.302(6)	0.319(6)	0.375(6)
BR-4 [4, 10]	0 164(4)	0 176(4)	0.123(2)	0 063(4)	0.068(4)	0.073(4)	0.082(4)	0.073(4)	0.082(4)
BR-5 [4, 6, 10]	0 741(4)	0 755(4)	0.502(4)	0 236(10)	0 285(10)	0.210(4)	0.247(4)	0.236(10)	0.285(10)

[a] Tabulated values accurate to approximately ±5 per cent.
[b] Numbers in square brackets indicate the stations at which lobes first appeared in the multibay stiffened cylinders. Numbers in parentheses are the stations at which the maximum e/h-values occurred, according to the method used.

TABLE 4 COMPARISON OF PREDICTED PRESSURES (PSI) FOR OCCURRENCE OF A VISIBLE LOBE—USING THE DIFFERENT METHODS FOR DETERMINING INITIAL OUT-OF-ROUNDNESS—WITH THE EXPERIMENTAL PRESSURES (PSI)

(1) Model no.	(2) Method (a) C.E.	Method (a) S.S.	(3) Method (b) C.E.	Method (b) S.S.	(4) Method (c) C.E.	Method (c) S.S.	(5) Expt.	(6) Col. (2)/Col. (5) C.E.	Col. (2)/Col. (5) S.S.	(7) Col. (3)/Col. (5) C.E.	Col. (3)/Col. (5) S.S.	(8) Col. (4)/Col. (5) C.E.	Col. (4)/Col. (5) S.S.
50	151	134	137	125	180	160	175	0.863	0.766	0.784	0.715	1.03	0.915
33	130	115	123	116	135	120	133	0.978	0 866	0.964	0.873	1.01	0.903
66	278	228	282	234	283	230	235	1.18	0.971	1.20	0.996	1.20	0.980
71	332	297	352	317	384	333	327	1.02	0.910	1.08	0.970	1.18	1.02
42	56	41.5	60	44	65	46	58	0.965	0 716	1.03	0.759	1.12	0.793
61	55	38	56	39	59	40 5	48	1.14	0.793	1.17	0.813	1.23	0.845
BR-1	74	63	75	67 5	98	80	80	0.922	0.788	0.938	0.844	1.22	1.00
BR-4	304	267	331	298	390	340	390	0.780	0.686	0.850	0.765	1.00	0.872
BR-5	57	50	68	61	92 5	76	80	0.713	0.625	0.850	0 763	1.16	0.950

of the shell The geometric ratios, yield points, experimental and theoretical collapse pressures for these cylinders are given in Table 2 The first six models in this table were tested some 20 years ago by Windenburg and Trilling (11), although they did not investigate theoretically the effect of initial out-of-roundness on the collapse pressure The last three models, which are multibay cylinders, have been tested recently at the Taylor Model Basin (13, 14).

A comparison of columns 6 and 8 in Table 2 shows a considerable discrepancy between the experimental and theoretical buckling pressures for even the simply supported cylinders For some models, it would also appear that axisymmetric yielding rather than buckling was the controlling mode of failure. This can be seen by comparing columns 8 and 10 in Table 2 However, it will be seen later when out-of-roundness is taken into account that the theoretical pressures for buckling-type failures are lower than the axisymmetric yield pressures. It is also of interest to note that Models 42 and 61 are the only models for which the experimental buckling pressures are higher than those predicted theoretically for simply supported cylinders, although the same supports were used for these two models as for the four models preceding them.

Using the geometric ratios and yield points shown in Table 2 and Equations [11] and [12] in Table 1, it is possible to construct curves showing the relation between the pressure at which yielding first occurs in the cylinder wall p_y, and the ratio of the initial out-of-roundness to the shell thickness e/h. Two such curves are shown in Fig. 2 for illustrative purposes. In constructing these curves, the value of m used in Equation [12] was the value of m which minimized Equation [11] in Table 1. These values of m are listed in parentheses in columns 8 and 9 of Table 2. These values of m do not actually give the minimum p_y for a given e/h. This point will be discussed later. It also should be noted that for $e/h = 0$ some of the curves in Fig. 2 do not attain the elastic buckling pressures tabulated in columns 8 and 9 of Table 2. When this is found to occur it means that the pressure to cause axisymmetric yielding of the shell is lower than the elastic buckling pressure.

In Table 3 are tabulated the maximum e/h-values obtained using the different methods for determining the initial out-of-roundness described earlier. The e/h-values were determined at mid-length for the unstiffened cylinders and at mid-length of the bays for the multibay cylinders. Two values are listed for each model under methods (a) and (c) because, in these methods, the number of lobes into which the perfect cylinder would buckle is used and this number is usually different for simply supported and clamped ends.

Now, selecting the experimental buckling pressures listed in column 6 of Table 2 in conjunction with their corresponding eccentricities listed in columns 2, 3, and 6 of Table 3, one can plot points on curves similar to Fig. 2 which represent values determined experimentally. In Table 4 we also give a numerical comparison of the theoretical and experimental pressures at which visible lobes first occur. The theoretical values in this table

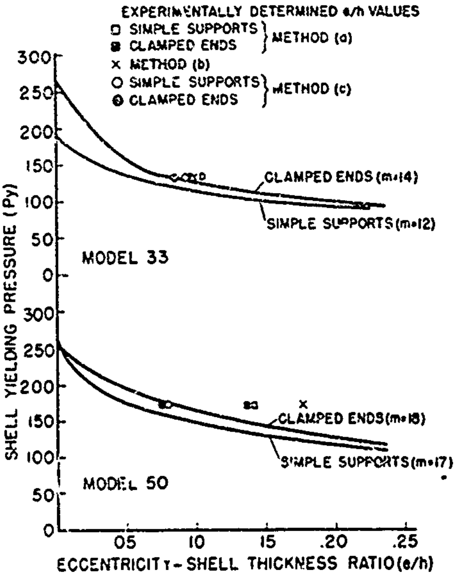

Fig 2 Shell Yielding Pressure for Models 33 and 50 Versus Eccentricity-Shell-Thickness Ratio

TABLE 5 AVERAGE VALUES OF COLUMNS (6), (7), AND (8) IN TABLE 4, CLASSIFIED ACCORDING TO CYLINDER TYPE

Cylinder type	Method (a) C.E.	Method (a) S.S.	Method (b) C.E.	Method (b) S.S.	Method (c) C.E.	Method (c) S.S.
Unstiffened	1 08	0 85	1 12	0 89	1 18	0 91
Stiffened	0 85	0 75	0 88	0 79	1 08	0.93

were obtained from the e/h-values listed in Table 3 in conjunction with the theoretical p_y versus e/h curves similar to Fig. 2. The last three columns in Table 4 show the ratios of theoretically predicted pressures for the occurrence of a visible lobe to those obtained experimentally, according to the various methods used for determining the initial out-of-roundness. The average values of these last three columns, classified according to whether the cylinders were stiffened or not, are tabulated in Table 5. It can be seen from Tables 4 and 5 that use of method (a), with the assumption of simply supported ends, was the most conservative in most cases and predicted pressures which were always below those obtained experimentally. However, the best correlation between the experimental results and the simplified theories discussed in this paper appears to be obtained when method (c) is used for determining the initial out-of-roundness and the cylinders are assumed to be simply supported

It was mentioned earlier that for a given e/h the value of m that would give the lowest p_y was not necessarily the value of m which minimized the expression for p_{cr} (Equations [11] in Table 1). It was also noted at that time, however, that the error obtained by assuming this to be so was not very great and also greatly reduced the computational labor. Some idea of the error involved can be obtained by referring to Fig 3. The curves in this figure are plots of m which minimize e/h for a given p_y. For a p_y of 80 psi it can be seen that the minimum value of e/h is 0 34 at $m = 18$, while at $m = 15$ (the value which minimizes p_{cr}) the value of e/h is 0 4. However, if we now fix the value of e/h at 0.4, then the minimum value of p_y is 75 psi and occurs at $m = 19$ Similar

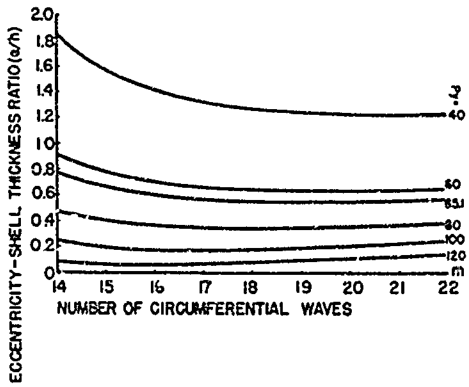

Fig. 3 Initial Eccentricity-Shell Thickness Ratio Versus Number of Circumferential Waves for Model BR-1 (Assumed Simply Supported)

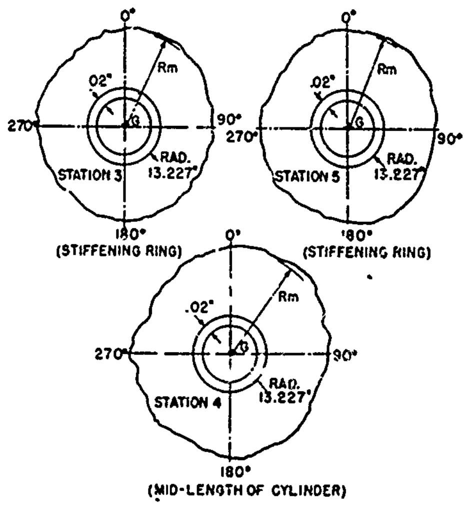

Fig. 4 Typical Initial Circularity Contours for Model BR-4

results are obtained with the other curves and also with the m versus e/h curves for clamped-end cylinders which we have not included here. It is thus seen that assuming the value of m which minimizes p_{cr} will also minimize p_y for a given e/h is slightly on the unsafe side, the magnitude of the error depending on the value of e/h taken However, owing to the fact that actual cylinders under hydrostatic pressure collapse in substantially the same number of lobes as is predicted by theory for the perfect cylinder, and also for simplicity in calculations, we have ignored this discrepancy.

It also will be remembered that in applying the analyses to stiffened cylinders we made the assumption that the stiffening rings were perfectly circular. This, of course, never occurs in practice Some idea of the degree of circularity actually present can be obtained by reference to Fig. 4. This figure shows the initial circularity contours of the stiffening rings bordering, and the shell at the center of one of the bays in which lobes first appeared in Model BR-4 If we adopt method (c) described earlier as our criterion for initial out-of-roundness, then the stiffening rings of this model had about one tenth the out-of-roundness of the shell. Similar results also were obtained for the other models. Thus, for the problem of inter-ring collapse of stiffened cylinders, the assumption of zero initial out-of-roundness of the stiffening rings appears to be a reasonable one.

It might be thought that a harmonic analysis of the initial out-of-roundness would show that the amplitude of the harmonic component corresponding to the value of m which minimized p_{cr} (Equation [11] in Table 1) was many times that of the other components. To investigate this point a harmonic analysis, using 72 subdivisions of the circumference, was made of the initial circularity contour at Station 4 of Model BR-4 using Filon's method (15) The number of waves into which a perfect cylinder with shell dimensions similar to BR-4 would buckle is, according to the linear theory used herein, 11 for simply supported ends and 12 for clamped ends From the harmonic analysis it was found that there were nine harmonics with amplitudes greater than the eleventh and two greater than the twelfth. It also was found that the amplitude of the largest harmonic ($m = 8$) was more than twice that of the twelfth mode and more than five times that of the eleventh mode Further, even this largest amplitude was much too small to effect a reasonable correlation between theory and experiment Thus the harmonic analysis of the BR-4 initial out-of-roundness contour did not produce any results which might be useful in practice.

In conclusion, it might be of interest to mention that some attempts have been made to determine experimentally the longitudinal form of the buckling displacemnts in the multibay cylinders The results of these few investigations are summarized in reference (14 However, more experimental work still remains to be done before any conclusions regarding the shape of the buckling displacement can be made

Summary

In the preceding sections an attempt has been made to explain some of the discrepancies that exist between experimental and theoretical results for cylinders subjected to external hydrostatic pressure To do this it was assumed that (a) the actual boundary conditions were somewhere between the extremes of simple supports and clamped ends, (b) the initial out-of-roundness was similar in form to one of the modes into which a perfect cylinder would buckle, (c) the stress distribution in the equilibrium problem for the perfect cylinder could be represented by the membrane stresses, and (d) the linear small-deformation equations of equilibrium would describe the problem adequately Any cold-working, residual, or welding stresses, or any elastic nonhomogeneity that might have been present were neglected It also was assumed that failure (formation of a lobe) would occur when the stresses at the most highly stressed point satisfied the octahedral shear-stress criterion Three simplified methods of measuring the initial out-of-roundness were also investigated The simplified analyses, together with the different methods of measuring out-of-roundness, were applied to nine steel welded cylinders with length-diameter ratios of [illegible] to 2, thickness-diameter ratios of 0 0025 to 0 [illegible] and yield points of the steel of 30,000 to 60,000 psi The correlation between experimental and theoretical results was quite good, when method (c) was used for determining the initial out-of-roundness of the cylinders

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Appendix

Approximate Equations of Equilibrium for an Initially Out-of-Round Cylinder

These equations have been derived previously by Bodner and Berks (3) using a different approach Our reason for this new derivation is to make clear what approximations and assumptions are involved in the simplified equations of equilibrium.

Consider a cylinder of mean radius R which has a small radial initial out-of-roundness w_0 Selecting x, θ, and z as co-ordinate axes and denoting the longitudinal, tangential, and radial elastic displacements by U, V, and W, it is then possible to show that the strains at the middle surface are given by

$$
\left.
\begin{aligned}
\epsilon_x &= U_x + \frac{1}{2}\,[V_x^2 + W_x^2] + W_x w_{0x} \\
\epsilon_\theta &= \frac{1}{R}\,[V_\theta - W] + \frac{1}{2R^2}\,[V + W_\theta]^2 + \frac{U_\theta^2}{2R^2} \\
&\quad + \frac{w_{0\theta}}{R^2}\,[V + W_\theta] + \frac{w_0}{R^2}\,[V_\theta - W] \\
\gamma_{x\theta} &= \frac{U_\theta}{R} + V_x + \frac{W_x W_\theta}{R} + \frac{W_x V}{R} - U_x V_x + \frac{U_\theta W}{R^2} \\
&\quad - \frac{U_\theta V_\theta}{R^2} + \frac{W_\theta w_{0x}}{R} + \frac{V w_{0x}}{R} + \frac{W_x w_{0\theta}}{R} + \frac{U_\theta w_0}{R^2}
\end{aligned}
\right\} \qquad [13]
$$

We shall simplify these complicated expressions to the following

$$
\left.
\begin{aligned}
\epsilon_x &= U_x + \frac{1}{2}\,W_x^2 + W_x w_{0x} \\
\epsilon_\theta &= \frac{1}{R}\,[V_\theta - W] + \frac{W_\theta^2}{2R^2} + \frac{W_\theta w_{0\theta}}{R^2} \\
\gamma_{x\theta} &= \frac{U_\theta}{R} + V_x + \frac{W_\theta W_x}{R} + \frac{W_\theta w_{0x}}{R} + \frac{W_x w_{0\theta}}{R}
\end{aligned}
\right\} \qquad [14]
$$

For the changes in curvature, we shall use the quantities previously used by Donnell (8) for perfect cylinders, viz.

$$
\chi_x = W_{xx} \quad \chi_\theta = \frac{W_{\theta\theta}}{R^2}, \quad \chi_{x\theta} = \frac{W_{x\theta}}{R} \qquad [15]
$$

The work done W_D by the uniform external pressure acting on all sides of the cylinder is given by the product of the pressure and the change in volume of the cylinder. Thus there follows (16)

$$
\left.
\begin{aligned}
W_D = p \int_0^{2\pi} \int_0^L \Big[& W - \frac{V_\theta}{2} - \frac{RU_x}{2} \\
& + \frac{2}{3}\,U_x(W + w_0) - \frac{1}{3}\,U(W_x + w_{0x}) \\
& - \frac{1}{3}\,U'(W_x + w_{0x}) + \frac{1}{3}\,U_x'(W + w_0) - \frac{W w_0}{R} \\
& - \frac{W^2}{2R} - \frac{V_\theta}{R}(W + w_0) - \frac{V^2}{2R} - \frac{1}{3}\,U_x V_\theta \\
& - \frac{1}{3}\,U V_{x\theta} - \frac{1}{6}\,U_x' V_\theta - \frac{1}{6}\,U' V_{x\theta} \Big] R\,dx\,d\theta
\end{aligned}
\right\} \qquad [16]
$$

where

$$
U' = \frac{\int_0^{2\pi} U\,d\theta}{2\pi}, \quad U_x' = \frac{\int_0^{2\pi} U_x\,d\theta}{2\pi}
$$

We shall simplify this complicated expression for W_D to the following

$$
W_D = p \int_0^{2\pi} \int_0^L \left[W - \frac{V_\theta}{2} - \frac{RU_x}{2} \right] R\,dx\,d\theta \qquad [17]
$$

It is also convenient to consider the U, V and W displacements to be made up of two parts

$$
\left.
\begin{aligned}
U &= \bar{U} + u \\
V &= \bar{V} + v \\
W &= \bar{W} + w
\end{aligned}
\right\} \qquad [18]
$$

where $\bar{U}$, $\bar{V}$, and $\bar{W}$ are the displacements which would occur in the equilibrium problem of a perfectly circular cylinder under uniform external pressure

The total potential of the system U_T is then obtained by adding the extensional and bending-strain energies of the shell and subtracting the work done by the external pressure. In calculating the extensional energy, we retained the terms in u, v, w, w_0 through the second order and the terms in $\bar{U}$, $\bar{V}$, $\bar{W}$ (directly proportional to the applied pressure) through the first order only; also, we neglected the effect of the deflection of the shell between supports on the displacements and stresses of the perfect cylinder.

Variation of U_T with respect to u, v, and w then gives the differential equations of the problem. Further manipulation of these equations results in the following

$$
\left.
\begin{aligned}
\nabla^4 u &= \frac{\nu}{R}\,w_{xxx} - \frac{w_{x\theta\theta}}{R^3} && \ldots \ (a) \\
\nabla^4 v &= \frac{(2+\nu)}{R^2}\,w_{xx\theta} + \frac{w_{\theta\theta\theta}}{R^4} && (b) \\
D\nabla^4 w + \frac{Eh}{R^2}\,\nabla^{-4} w_{xxxx} &= p + h\Big[\bar{\sigma}_x(w + w_0)_{xx} \\
&\quad + \frac{2\bar{\tau}_{x\theta}}{R}\,(w + w_0)_{x\theta} + \frac{\bar{\sigma}_\theta}{R^2}\,(w + w_0)_{\theta\theta}\Big] && (c)
\end{aligned}
\right\} . \quad [19]
$$

where $\bar{\sigma}_x$, $\bar{\sigma}_\theta$, and $\bar{\tau}_{x\theta}$ are the membrane stresses which would occur in a perfectly circular cylinder and which were assumed to be constant in deriving the foregoing equations.

For the case of uniform external hydrostatic pressure applied on all sides of a perfectly circular cylinder, and neglecting the deflection of the shell between supports, there results

$$
\bar{\sigma}_\theta = -\frac{pR}{h}, \quad \bar{\sigma}_x = -\frac{pR}{2h}, \quad \bar{\tau}_{x\theta} = 0 \qquad \ldots [20]
$$

Substituting the Relations [20] into Equation [19c] yields

$$
\begin{aligned}
D\nabla^4 w + \frac{Eh}{R^2}\,\nabla^{-4} w_{xxxx} + pR\Big[\frac{1}{2}\,(w + w_0)_{xx} \\
+ \frac{1}{R^2}\,(w + w_0)_{\theta\theta}\Big] - p = 0 \quad \ldots [21]
\end{aligned}
$$

Now define a stress function of the total membrane stresses F, such that

$$
\frac{N_x}{h} = \frac{F_{\theta\theta}}{R^2}, \quad \frac{N_\theta}{h} = F_{xx}, \quad \frac{N_{x\theta}}{h} = -\frac{F_{x\theta}}{R} \qquad . \ [22]
$$

Using Equation [14] and the stress-strain relations it is then easy to show that

$$
\begin{aligned}
\nabla^4 F = E\Big\{ & -\frac{1}{R}\,W_{xx} + \frac{W_{x\theta}^2}{R^2} - \frac{W_{xx}W_{\theta\theta}}{R^2} \\
& + \frac{2W_{x\theta}w_{0x\theta}}{R^2} - \frac{W_{xx}w_{0\theta\theta}}{R^2} - \frac{W_{\theta\theta}w_{0xx}}{R^2}\Big\} \qquad . \ [23]
\end{aligned}
$$

As we are interested here in the linear problem, and as we have previously neglected the deflection of the shell between supports, Equation [23] reduces to

$$
\nabla^4 F = -\frac{E}{R}\,w_{xx} \qquad [24]
$$

Equations [19a], [19b] [21], and [24] together with the appropriate boundary conditions define the problem.

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